

## Optimization of the sustainability profit generated by the production of beams

TOMAŽ ŽULA & STOJAN KRAVANJA

**Abstract** The paper presents the optimization of the sustainability profit generated by the production of simply supported beams in the area of civil engineering. A number of beams are proposed to be designed from three different material alternatives: from the structural steel, from the reinforced concrete and from the laminated timber. For this reason, three optimization models of beams are developed for the three materials. In addition, two different objectives are defined for each different material alternative: for the economic profit and for the sustainability profit (which includes eco costs of the global warming). The proposed objective functions are subjected to the design, resistance and deflection constraints of the beams, determined in accordance with Eurocode 2, 3 and 5 specifications. The optimizations of the beam alternatives are performed by the mixed-integer non-linear programming (MINLP) approach. GAMS/Dicopt is used. The task of the optimization is to find the most advantageous material alternative for the beams. The numerical example, presented at the end of the paper, clearly shows that the reinforced concrete beams exhibit the highest economic profit, but the timber beams give the highest sustainability profit.

**Keywords:** • Sustainability profit • GHG emissions • Structures • Mixed-integer non-linear programming • MINLP •

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## 1 Introduction

The paper handles with the optimization of the sustainability profit generated by the production of simply supported beams in the area of civil engineering. In this case sustainability profit is a summation of the economic profit and eco costs of the global warming. A number of beams are considered to be designed from three different material alternatives: from the laminated timber, from the structural steel and from the reinforced concrete. The objective of this paper is to find the optimal design of the simply supported beam subjected to the highest economic profit and to the sustainability profit, performed by mixed-integer non-linear programming approach.

In the fields of optimization and sustainability, different optimization techniques and objectives have been proposed. Zaforteza et al. (2009) used simulated annealing algorithm (SA) applied to two objective functions, namely the embedded CO<sub>2</sub> emissions and the economic cost of reinforced concrete structures. Camp and Huq (2013) have proposed a hybrid big bang-big crunch algorithm (BB-BC) for the optimal design of reinforced concrete frames. The objective was to minimize the total cost or the CO<sub>2</sub> emissions. Alonso and Berdasco (2015) presented the carbon footprint of sawn timber products. Li et al. (2017) have introduce a topology optimizer to get the best-possible welded box-beam structures that emit less greenhouse gases by using improved ground structure method (IGSM).

## 2 MINLP model formulation

Since the problem of simply supported beam is the non-linear discrete-continuous optimization problem, the MINLP is applied for the solution. The general MINLP optimization problem can be formulated as follows:

$$\begin{aligned} \min \quad & z = f(\mathbf{x}, \mathbf{y}) \\ \text{subjected to: } & g_k(\mathbf{x}, \mathbf{y}) \leq 0 \quad k \in K \\ & \mathbf{x} \in X = \{\mathbf{x} \in R^n: \mathbf{x}^{LO} \leq \mathbf{x} \leq \mathbf{x}^{UP}\} \\ & \mathbf{y} \in Y = \{0, 1\}^m \end{aligned}$$

where  $\mathbf{x}$  are the continuous variables and  $\mathbf{y}$  are the discrete (0, 1) variables. Function  $f(\mathbf{x}, \mathbf{y})$  is the objective function for the economic profit and for the

sustainability profit (which includes eco costs of the global warming).  $g_k(\mathbf{x}, \mathbf{y})$  stands for the design, resistance and deflection constraints.

### **3 Numerical example**

The example shows the optimization of 500 equal simply supported beams. Each beam is 9.0 meters long, subjected to the combined effect of the dead-weight, the permanent continuous load of 9.0 kN/m ( $g$ ) and the imposed variable continuous load of 9.0 kN/m ( $q$ ), see Fig. 1.

Each simply supported beam is proposed to be made from three different material alternatives: from the laminated timber, from the structural steel and from the reinforced concrete. At this point the comparison and the competitiveness between these three materials of the beams was investigated for various material and dimension alternatives, and for two different objectives i.e. for the optimization of the economic profit and of the sustainability profit.

For comprehensive topology optimization problem, we usually use program MipSyn (Kravanja, 2010). As the optimization problem of the beam a simple discrete and non-linear problem, Dicopt (Grossmann, 2002) was selected for application. Six optimization models for the simply supported beam (SIMSBOPT) were developed as a combination between three different materials (timber, steel and concrete) and two different objective functions. For mathematical modelling GAMS (General Algebraic Modelling System), (Brooke et al., 1988), was used. The models consist of the objective functions, subjected to the design, loading and resistance constraints known from structural analysis. The dimensioning and deflection constraints were performed according to Eurocode specifications: Eurocode 5 (2004) for timber, Eurocode 3 (2005) for steel and Eurocode 2 (2004) for the reinforced concrete. The beams were checked for the shear, bending moment and lateral torsional buckling resistances as well as for the vertical deflections.

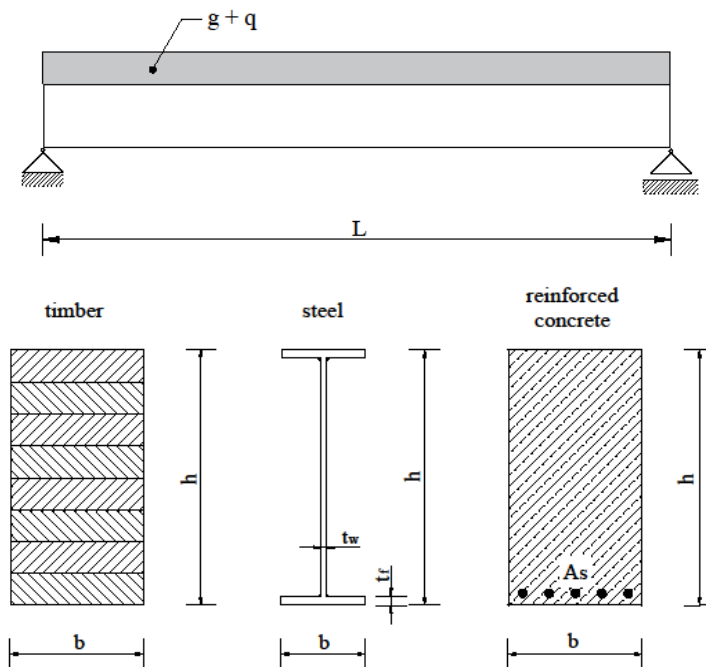


Figure 1: Simply supported beam

The simply supported beam superstructure comprises three different materials. The laminated timber beam superstructure comprises 101 different rounded dimension alternatives for the cross-section width and 131 rounded dimension alternatives for the cross-section height. The steel beam superstructure includes 3 different steel grades, 8 different dimension alternatives of the standard steel plate thicknesses for flanges and webs separately, 1051 rounded dimension alternatives for the width of the flange and 1301 rounded dimensions alternatives for the height of the web. In addition, 7 different concrete grades, 13 standard reinforcing steel bars, 131 rounded dimension alternatives for the cross-section height and 101 rounded dimension alternatives for the cross-section width (rounding up on whole centimeters) are involved in the reinforced concrete beam superstructure.

The given material and dimension alternatives (binary variables) gives 13231 structure alternatives for the timber beam, 262 531 392 different structure alternatives for the steel beam, and 1204021 different structure alternatives for the reinforced concrete beam.

Two different objective functions were proposed for two different defined criteria. The first criterion of the optimization includes the maximization of the economic profit ( $P_E$  [€]) of 500 equal beam structures. The economic profit is determinate as a sum of the selling price, the self-manufacturing material and labor costs, and overheads. The objective function was defined for three different materials separately, see Eq. (1).  $N$  is a number of simply supported beams ( $N = 500$ ),  $C_S$  [€] is a selling price of a single simply supported beam,  $C_{M_i}$  [€/kg] represents the material unit prices of ( $i \in I$ : laminated timber, impregnation and protection paint for the timber beam; structural steel, electrodes, gas consumption and anticorrosion-resistant paint for the steel beam; and concrete, reinforcing steel bars and formwork slab-panels for the concrete beam).  $\rho_i$  [kg/m<sup>3</sup>] is the corresponding unit mass and  $V_i$  [m<sup>3</sup>] is volume. While  $C_{L_j}$  stands for the hourly labor costs [€/h],  $t_j$  [h] are times required for ( $j \in J$ : impregnating and painting the timber beam; cutting, welding and painting the steel beam; and placing, curing and vibrating the concrete, cutting and placing the reinforcement, and paneling the concrete beam), and  $f_O$  is an indirect cost factor for overheads ( $f_O = 2$ ). More detail about cost items used in the economic objective function see (Jelušič, 2017) and (Kravanja, 2017).

$$\max P_E = N \cdot (C_S - C_{M_i} \rho_i V_i - C_{L_j} t_j f_O) \quad (1)$$

The second criterion is the maximization of the sustainability profit ( $P_{SUS}$  [€]), calculated for 500 beams as a summation of the economic profit and eco costs of the global warming (EVR, 2018) caused by the beam production. The objective function was defined for three materials separately, see Eq. (2).  $C_{GW}$  (€/kg CO<sub>2</sub> eq.) is a price of global warming, 0.116 €/kg CO<sub>2</sub> eq. (EVR, 2018),  $\rho_k$  [kg/m<sup>3</sup>] and  $V_k$  [m<sup>3</sup>] are the corresponding unit masses and volumes, respectively and  $f_{CFEF_k}$  is carbon footprint emission factor ( $k \in K$ ; for the timber beam, steel beam and for the reinforced concrete beam). The carbon footprint emission factor used in the study are 0.69 kg CO<sub>2</sub> eq./kg for timber, 1.72 kg CO<sub>2</sub> eq./kg for steel, 0.11–0.16 kg CO<sub>2</sub> eq./kg for concrete and 2.43 kg CO<sub>2</sub> eq./kg for the reinforcing steel bars.

$$\max P_{SUS} = P_E + N \cdot (-C_{GW} f_{CFEF_k} \rho_k V_k) \quad (2)$$

Table 1 shows the results of the optimization for three different materials and two different objective functions. The obtained results show that the concrete

beams exhibit the highest economic profit while the laminated timber beams show the highest sustainability profit. The steel beams exhibit the worst results in all three criteria.

**Table 1: Results of the simply supported beam optimizations**

Criterion		Timber GL24h	Steel S 235	Reinforced Concrete C 50/60
1.	<b>Economic profit (€)</b>	<b>123 890</b>	<b>-192 801</b>	<b>161 939</b>
	b (cm)	21.0	32.9	29.0
	h (cm)	77.0	50.0	59.0
2.	<b>Sustainability profit (€)</b>	<b>97 179</b>	<b>-257 776</b>	<b>93 745</b>
	b (cm)	21.0	32.9	29.0
	h (cm)	77.0	50.0	59.0

1. Economic profit; 2. Sustainability profit

## 4 Conclusion

The paper presents the optimization of the sustainability profit generated by the production of simply supported beams in the area of civil engineering. The optimal solutions are calculated using two different objective functions, i.e. economic profit and sustainability profit. The optimizations of the beam alternatives are performed by the mixed-integer non-linear programming (MINLP) approach. The numerical example clearly shows that the reinforced concrete beams exhibit the highest economic profit, but the timber beams give the highest sustainability profit.

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