Exploring Early Childhood Teachers' Differentiation Practices in Teaching Mathematics with Learning Trajectories

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Abstract Teachers in preschools work with children with various ability levels; therefore, they should be ready to utilize differentiated instruction. However, they are not well equipped to meet the learning needs of children with individual differences when teaching mathematics. Research indicates that Learning Trajectories can help teachers when serving children with various learning needs. In this study, five preschool teachers participated in a Learning Trajectory based PD program for ten months to teach young children mathematics better. This research reports on the interviews of the teachers about their differentiation strategies. The content analysis of the interviews revealed that the teachers used the LT to accommodate their instruction to each child's level of understanding. They also utilized diverse ways of grouping children and they considered children's feelings. The results indicate that the LT can be a great tool in helping teachers differentiate their instructions along with sensitive and caring classroom culture.

Keywords: differented instruction, early mathematics, learning trajectories, professional development, teacher education

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1 Introduction

High quality educational standards in pre-school requires teachers to be prepared for supporting students with diverse educational needs (Garvis et al., 2022) because each child comes to school with a different background, including different genetic characteristics, interests, family effects and cultural backgrounds (National Association for the Education of Young Children, 2019). In addition, gaps among children may be higher if children are coming from families with diverse socio-economic backgrounds which strongly affects the development of the children (Kulic et al., 2019; Rindermann & Baumeister, 2015). Hart and Risley (1995) reports that when certain risk factors are present, it is possible to see differences between children's cognitive development as early as 18 months. Therefore, it is inevitably necessary for teachers to know how to differentiate teaching in preschool. However, that is not always the case, especially in mathematics.

Historically, mathematics has not been a popular subject in early childhood education; therefore, preschool teachers are not well equipped and struggle with teaching young children quality mathematics (Lindmeier et al., 2020; Sarama & DiBiase, 2004). Additionally, the situation becomes more complicated when teachers work with children who are gifted or have special needs to whom teaching requires certain adaptations. At this point, we believe that Learning Trajectories (LT) can guide teachers in the education of children with varying levels of developmental skills because the necessary conditions for implementing differentiated instruction can be easily provided by using the LT (Clements & Sarama, 2021).

According to Tomlinson (1999), who writes extensively about meeting individual learning needs, differentiated instruction requires teachers "to engage students in instruction through different learning modalities, by appealing to different interests, and by using varied rates of instruction along with varied degrees of complexity (p. 2)". This task should be challenging for teachers when teaching mathematics as they need to have a profound understanding of the content, pedagogy, and children's thinking in all areas of early mathematics (Carpenter et al., 1996; Gasteiger & Benz, 2018). Differentiated Instruction also entails developing and employing modifications in content, process, and product domains of curriculum (Tomlinson, 1999). However, literature addresses the inferior quality of teachers' modifications to support children with diverse needs (Kurth et al., 2012; Strogilos et al., 2018; Ware

et al., 2011); thereby indicating the demand for new ways of supporting teachers in their efforts to differentiate instruction.

LT can be practical in that sense because teachers who learn about and implement LT understand the nature of the primary mathematics content better. Additionally, the nature of LT, where math skills are ordered from the most basic to the most complex is appropriate for facilitating learning for all children (Baroody et al., 2022; Sarama & Clements, 2009). It lubricates the process of defining students' level of content knowledge in all areas of mathematics and guides teachers to level up students' thinking (Sarama et al., 2022). Besides its use as an assessment tool, the LT also guides the curriculum and instruction (Confrey et al., 2014; Daro et al., 2011). In their research with in-service and pre-service elementary teachers, Wilson and his colleagues (2013) studied teachers' use of learning trajectories for rational numbers to develop their understanding of student thinking. The researchers found that using learning trajectories was effective in helping teachers form models of student thinking and shape their understanding of mathematics and student thinking (Wilson et al., 2013). Another research (Sarama et al., 2017) reported that early childhood teachers who used learning trajectories in teaching early mathematics improved their noticing of student thinking. That improvement in teacher noticing is likely to support teachers in designing learning contexts and activities more likely to facilitate children's mathematics learning. Since the LT includes skills from the most basic to the most complex, the teachers can quickly determine a baseline for each child. The teachers who monitor their students' progression via the LT would make instructional decisions easily for individual children (Sarama et al., 2017). Therefore, the LT naturally turns into a differentiated instruction tool. Teachers' expectations of children's potential are also likely to increase when they use the LT in teaching mathematics. Clements and Sarama conclude, "teaching with learning trajectories is the best way to address the needs of all children, especially those with special needs" (2021, p.348). Yet, we do not know much about how early childhood teachers using the LT for teaching mathematics differentiate their instruction. Also, literature suggests that there is little known about early childhood teachers' practices concerning modifications to differentiate their instructions(DeBaryshe et al., 2009; Purcell & Rosemary, 2008; Strogilos et al., 2018; Ware et al., 2011). Therefore, researchers call for new studies to learn more about the nature of the subject, especially small-scale studies investigating different methods of differentiation in early childhood classrooms (DeBaryshe et al., 2009). The current study will

contribute to the field by providing inside voice from the classrooms where the LT based mathematics program took place.

1.1 Research Questions

The following questions guided the present research:

- How did the teachers who participated in the LT-based PD program on teaching early mathematics implement differentiated instruction in their classrooms?
- What challenges did the teachers face while differentiating the instruction in their classrooms?

2 Methods

In this qualitative research, we worked with five early childhood teachers to explore the above questions.

2.1 Setting

The purpose of this research is to study a group of early childhood teachers' differentiation practices in teaching mathematics. Five teachers who participated in a Learning Trajectories Based (LT-Based) Professional Development (PD) program on teaching early mathematics instruction put what they learned at the PD into practice in their classrooms. While they were teaching mathematics aligned with the PD program principles, they found themselves to accommodate students with individual differences. The teachers needed to differentiate the content to adapt their instruction to diverse educational needs. In this paper, we report how they differentiated the mathematics instruction.

2.2 Teachers

Five female teachers working at a private kindergarten voluntarily participated in the current research and gave their informed consent in writing to take part in this project. Although all teachers were required to attend the LT-Based PD program, they all participated in the research section at their own will. Collected personal data

was managed according to the related laws and regulations. All teachers received education in child development. As seen in Table 1, three of them own 4-year university degrees, one of them owns a 2-year university degree and one teacher holds a vocational school degree (equivalent of high school) in child development. The teaching experiences of the participants range from three to fifteen years (Table 1).

Table 1: Demographics of the participants with class age and size

Participant pseudonyms	Experience	Education	Class age; size
Teacher 1	10 years	4-year university	5 yrs. old; 15 children
Teacher 2	15 years	Vocational school	5 yrs. old; 16 children
Teacher 3	3 years	4-year university	4 yrs. old; 8 children
Teacher 4	4 years	2-year university	4 yrs. old; 9 children
Teacher 5	4 years	4-year university	3 yrs. old; 15 children

In addition to their formal education, the teachers attended a wide range of professional development programs and seminars; yet it was the first time they participated in a PD program on early mathematics instruction. Their mathematics and mathematics teaching knowledge were extremely limited. They received at most one or two mathematics-related courses at college level.

As seen in Table 1, the school serves children from different age groups. During the 10-month implementation of the PD program sixty-three children received early math instruction from five teachers. The children were from middle-class families in their region. Most of the parents were college-graduates working at white-collar jobs. In many cases, both parents were working during the days.

2.3 Professional Development (PD) Program

2.3.1 Overview

The authors of this research were invited by a private school located in a large metropolitan city of Türkiye to design and deliver a PD program on teaching mathematics to young children. While one of the teacher educators is an early childhood educator, the other is a mathematics educator. Both researchers have extensive previous experience in conducting PD programs on early mathematics

teaching. The PD efforts began at the beginning of September 2021 and lasted until the end of June 2022. Thus, altogether the present research reports about early childhood teachers' experiences in a 10-month PD program.

2.3.2 Purpose

The main purpose of the program was to enhance teachers' understanding and implementation of mathematics instruction for children between 3 - 6 years old. Among other frameworks (Confrey & Maloney, 2010; Simon & Tzur, 2009; Sztajn et al., 2012; Wickstrom & Langrall, 2020), The Learning and Teaching with Learning Trajectories (LT) approach for early mathematics (Clements & Sarama, 2017, 2021) guided the activities of the PD. The LT consists of three related components: a) a mathematical goal, b) a hypothetical developmental progression through which the child will move forward to reach the goal and c) a set of instructional activities that are supposed to help the child to move from one level of thinking to a more complex level (Sarama et al., 2016). Based on a rich body of research in mathematics education, developmental psychology, and cognitive psychology (Sarama & Clements, 2009), Clements and Sarama identified mathematical skills that children should acquire to reach the goal (2021). The skills are ordered from the most basic to the most complex and are observable and easy to assess.

2.3.3 Content

The entire program was devoted to teaching counting, subitizing, comparing numbers, adding, and subtracting, composing numbers to young children. Teacher educators introduced the content of each topic with the associated skills as outlined in the LT framework (Clements & Sarama, 2017). For each mathematics topic, a table with a list of the skills was handed out to teachers so that they could use the skills as an assessment and curriculum guide.

Teachers' approach to children was also critical in the PD program, where the aim was to support children's learning on the one hand and to reinforce their self-confidence on the other. LT-based list of skills guided teachers to find the appropriate level of activities where each child was challenged; yet they successfully solved the problem. This strategy was critical to feeding children's self-confidence, especially struggling children. The motto was "No child will leave the activity without

any feeling of success." Therefore, teachers were encouraged to find ways to improve children's learning by offering challenging tasks and utilizing many ways to scaffold in case they need assistance.

2.3.4 Implementation

The teacher educators and teachers met weekly to discuss how to teach mathematics concepts and procedures. Each face-to-face meeting took about one-hour on Thursdays after the school day had ended. Between the face-to-face sessions the teachers could communicate with the teacher educators via phone and WhatsApp, a mobile instant messaging (MIM) tool. They asked their questions regarding the content of the PD program and mathematics concepts. Additionally, they shared videos and pictures of mathematics teaching episodes to receive feedback from the teacher educators. They received feedback on their videos via WhatsApp and at the face-to-face meetings.

At the weekly meetings, the teacher educators shared essential components of the mathematics topics. For example, they explained principles of counting (Brownell et al., 2014), perceptual and conceptual subitizing (Clements, 1999) and problem types for addition and subtraction (Fennema et al., 1996). The LT for the mathematics topics were introduced to the teachers at the meetings. The teacher educators reviewed the table with a list of skills associated with the relevant mathematics content (LT Table). They all carefully studied each of the skills so that they could effectively understand how to observe and teach the skills. The teacher educators and teachers offered relevant teaching ideas. The teachers were encouraged to use the skills table for the assessment of children and delivering the content. Additionally, videos of actual classrooms were viewed and discussed regularly. It was a big topic on the agenda of the weekly meetings to give feedback and discuss videos. Usually, one or two videos for each teacher were discussed at a meeting.

The school administrator attended all the weekly meetings in-person and took note of everything discussed during the meeting. She also visited the classrooms regularly to give feedback to the teachers and aided them if necessary. The school administrator was highly knowledgeable about the content of PD. The researchers, teacher educators, visited about three times a month. They engaged in conversations with the administrator regarding the classroom implementations. The researchers

were answering the administrator' questions about specific aspects of the implementation such as how to use mathematics manipulatives or how to lead successful mathematical talk with children. The teachers and administrator reported that the administrator was supporting the teachers regarding mathematics instruction via WhatsApp or individual interactions. As a result, the administrator acted as an on-site coach.

2.4 Data Sources

The teachers were individually interviewed twice by one of the teacher educators who has extensive experience in conducting research interviews. The first interview was conducted about in the middle of the PD program, January 10th, and the second interview was conducted at the end of the program, the last week of June. The teachers were also interviewed informally at the weekly meetings to ask about their reactions about the program and their experiences. The purpose of the interviews was to explore teachers' reflections on the implementation of the PD, their professional growth, and their classroom implementations, including differentiation practices. In this paper, we only focused on how they differentiate their instruction to reach students with diverse needs. In the interview protocols, we added questions which were solely specific to differentiation. We asked them the following questions regarding their differentiation practices:

- Were there any situations where the children had difficulties during the implementation phase?
- What do you do for slow learners?
- How did you assess whether children learned or not?

2.5 Data Analysis

This research was carried out by using qualitative content analysis. Content analyzing process includes the coding, categorizing, and naming the data (Miles & Huberman, 1994; Sharan B. Merriam, 1994). Interviews were transcribed and read by all researchers several times. Most relevant and significant expressions for the aim of the study are identified and occurred as codes. Categories are decided by grouping all codes in accordance with their similarities and differences. Five categories emerged and were named as following: Planning instruction appropriate for the

child's level, grouping-pairing, individual work with the child, considering the child's feelings and using LT as a differentiation tool. For reliability issues, after reading the data, researchers discussed occurring codes and reached a consensus on themes and sub-themes.

3 Results

In this section, the findings obtained from the data collected through interviews with 5 teachers participating in the research are included. Differentiation practices used by early childhood teachers in teaching mathematics are grouped into five main themes: Planning instruction appropriate for the child's level, grouping-pairing, individual work with the child, considering the child's feelings, and using LT as a differentiation tool. While explaining the themes, the sub-themes identified for each theme and the opinions of the teachers were included.

3.1 Planning appropriate instruction for the child's level

The first theme emerging from the data was teachers' concern for providing appropriate level of instruction when teaching mathematics. Three sub-themes occurred related to this theme: Adjusting implementation, slowing the process down, and progressing after the child reaches the previous level on LT.

Teachers increased or decreased the cognitive demand of activities according to child's level of ability. For instance, Teacher 1 mentioned that she used larger numbers in mathematics games and more challenging questions when she was working with a gifted child.

Similarly, another teacher, Teacher 2, commented,

"My table was small for the activity with cards up to a hundred, but the gifted girl was ready to count to three-digit numbers. (...) Activities up to a hundred really seems so easy to some children."

Teacher 2 prepared extra counting cards for the gifted child, who could count to three-digit numbers, so that the child would extend her counting skills. As expected, the teachers also had children who were behind their classmates. Teacher 4 talked

about how she tried to meet the needs of a group of students with diverse levels of learning.

She commented,

"…I asked questions with larger numbers when I work with children who are above the class average. For example, when I noticed that two of my students had higher learning capacities, I challenged them in mathematics. Yet, I ask questions with slightly smaller numbers when I work with others, considering their level. So, they do not feel bad about themselves."

In addition to making the activity easier, teachers also slowed their teaching process down for children whose learning speed requires a slower course.

It was also found that the primary tool that teachers used for differentiation was the LT. They followed the activities within the scope of LT considering the children's individual needs. Their use of the LT is a direct reflection of what they learned in the PD. All teachers stated that when they worked with a child who had difficulty reaching a specific outcome, they returned to the previous level in the LT framework. They practiced making him master the necessary skills he had difficulty with. After reaching the previous level, the child's participation in activities that moved him forward was ensured.

After assessing children based on their correct responses, the teachers determined children's level on the learning trajectories for counting. They were expected to base the instruction on the assessment results. The teachers reviewed the learning trajectories to have a general idea about the children's overall level. Following the general overview, they determined whether there were striking differences among individual children. The teachers were encouraged to consider both the general class level and individual differences to better adjust the instruction. For instance, if a group of children shows counting skills up to 10, the teacher aimed at the counting skills beyond 10 to further student learning. Additionally, if a child were behind their peers, the teacher would be designing activities that require counting within ten. Furthermore, the teachers were expected to increase the demand of the activities for children who are ahead of their peers.

3.2 Grouping-pairing

Some teachers said they used grouping and pairing methods sometimes for differentiation. They shared that they paid attention to the children's thinking level while grouping them in activities requiring small group work.
Teacher 4 commented,

"… to establish a balanced learning atmosphere, I mix up children by their levels of reaching mathematical goals. (...) Children can look at each other and learn something from each other because it really helps."

Teacher 4 and other teachers included children ahead and behind the general class level as much as possible in the same group. They encouraged the children to learn from each other by supporting their interactions. While doing this, they aim to ensure that the children behind the class do not sit together and that their learning motivation does not decrease by feeling unsuccessful.

Teacher 2 commented,

"I group children in a way that children do not feel bad for not being competent enough. Because when children who are behind others always come together, their emotional mood goes down, unfortunately."

Teacher 2 also stated that after grouping and pairing students to support peer learning, she gave instructions encouraging children to interact, give each other clues and help each other to make the learning climate more welcoming and natural.

Another teacher, Teacher 1, stated that she especially grouped the children far ahead of the general class level in particular activities. She explained that children could easily follow each other's actions and learn from each other during the activities. Also, it was practical to bring together children who perform exceptionally well to prevent others from feeling incompetent.

3.3 Individual work with the child

Working individually was often necessary when teachers worked with diverse groups. For instance, it was not always possible to evaluate whether the child with individual needs had achieved the relevant goal when working with a large group.

Teacher 3 explained,

" I cannot understand whether he/she understood any concept when working as a large group. In large groups, children who are behind their peers have the potential of being invisible."

As emphasized by Teacher 3, children needed to be evaluated individually. Teacher 3 further explained that, in large groups, they could copy each other's answers. Teacher 1's comments added more on the subject that it was necessary to work one-on-one with children in need to prevent them from moving to the next level without fully comprehending.

Another reason for working individually was the child's need for individual instruction since he/she could not learn in groups. All but one teacher stated that they conduct individual studies with children with individual needs to provide mathematics instruction appropriate for children's individual needs. They mentioned using different strategies, including staying at the proper level of LT until the child learns, giving extra time, repeating, making the child sit near the teacher, and doing individual work to check whether the child has learned.

Teacher 1 stated that she spared time for a child she knew needed more time during the activities. Although the rest of the class started to work on the next goal, they continued to work on the last goal with the child. Teacher 1 did not expect the child to move to the next level if she decided that was unnecessary. Another teacher, Teacher 4, stated that she did extra individual work for the children who were behind the average class level regarding the learning goals. She repeated the same activity for children in similar situations many times.

Teacher 2 used another strategy; she commented,

"I take them (children behind the general class level) near me. Starting with individual work with smaller numbers. I'm taking it down, slowing it down."

As data shows, the participant teachers worked with children with special needs individually when they evaluated those children's level of mathematical understanding. Also, they offered individual instruction when those children could not follow the instruction in groups.

3.4 Considering the child's feelings

Throughout the interviews, all teachers consistently emphasized the importance of considering children's feelings while planning mathematics instruction appropriate for individual children. They rationalized their differentiated educational activities with the need to improve children's moods or to prevent them from feeling bad. They reiterated that children cognitively behind their peers should feel accepted, successful, and confident in their classrooms.

For instance, Teacher 4 commented,

"At the end of the activity, I want them to have good feelings. Each child should say, "I can do it!"

Another teacher, Teacher 2 stated that it was important not to force children to do things beyond their ability; instead, teachers should focus on what they could do. Otherwise, she said, they were likely to feel bad and insufficient; these children need encouragement.

Teacher 2 explained further and gave an example:

"... while one of my children was doing subtraction, I said, 'you are doing very well!' He got enthusiastic and said, 'I'm very good at subtraction!' He was so excited and said, 'I'm already very good at subtraction' all day."

She stated that this encouraging and motivating approach improved the child's attitude towards mathematics and self-confidence. The teachers used the above strategies to comfort them, make them feel accepted, and support their

development. Yet, they sometimes needed professional help. Then, teachers referred them to psychological counseling services in their school.

Teacher 1 gave an example,

"One of my students is shy and fearful of numbers. First, I wanted to relieve that fear and anxiety. I talked about him to the school counselor. Is he having trouble understanding or paying attention? There is a fine line there. The counselor is an expert on this subject, of course."

Teacher 1, as stated, received support from experts and focused specifically on the child's development process.

3.5 Using LT as a differentiation tool

As interviews were analyzed, the findings suggested that the teachers actively used the LT Table as a differentiation tool. First, they determined each child's stage by conducting individual interviews. What each child could do was marked with "+" and what they could not do with "-". Based on these assessments, teachers made plans to improve the skills in which children got "-".

After describing her way of using LT Table, Teacher 4 said,

"For example, I don't try to teach place value concepts, including ones and tens, to a child who cannot count."

Utilizing the LT Table enabled teachers to assess students based on factual data and determine the general class level and the levels of individual children on the same document. The teachers reported that they noticed the children who needed individual support better.

Teacher 1 commented,

"LT checklist tool clearly reveals whether a child needs individual support and our starting point."

Similarly, Teacher 4 said,

"LT is our map; it is our road map."

When we asked how often and how they used this tool, teachers said they reviewed it every week, at varying intervals for each child, and evaluated the students they thought progressed and turned the "-" into "+". Teachers emphasized the importance of not interrupting children's play using LT Table.

In this regard, Teacher 1 said,

"Not interrupting the child's play but attending to the child's play is not a waste of time, it is a necessity."

Teachers stated that they prepared game-based mathematics activities with appropriate materials to improve the skills that the child acquired "-". The children were very enthusiastic about participating in the related activities.

Teacher 2 stated that children consider assessment as playing one-on-one games with the teacher and added,

"The games and stations we set up on the table (for assessment) are interesting. The children feel special, and they say, 'When will you pick me up?'"

Therefore, the data showed that the teachers' use of games and plays as a part of their assessment with LT Table improved the tool's efficiency.

4 Discussion and Conclusion

Research results show that teachers make many modifications, especially in the context of process and content. Some strategies teachers employed were organizing small group and individual activities, pairing students with varying levels of understanding, increasing and decreasing the difficulty levels of activities, giving extra time, repeating, using different materials, and slowing down. These modifications were similar to the ones utilized by the teachers who participated in other studies (Deunk et al., 2018; Strogilos et al., 2018). In addition, two issues stood out in this study. The first is that teachers actively used LT as a differentiation tool. In a similar study conducted before, Debaryshe and colleagues (2009) reported that the participating teachers did not assess student progress as often as required, and they perceived this process as "too much of a burden" (DeBaryshe et al., 2009). On the contrary, the current study participants did assessment activities regularly. Indeed, the participating teachers were not told how often they would evaluate; they

were only asked to follow the children with the LT Table. They decided how often they would use it. Possible explanations for this result may be as follows: The study was conducted with a small group where the role of personal relationships might have motivated teachers. The two researchers held regular face-to-face meetings each week and gave individual feedback to the teachers. They reviewed the teachers' work and shared the points they liked or needed to be corrected. Group dynamics might have motivated the teachers. Another explanation might be the practical nature of the LT Table. During the training sessions, the researchers observed that the LT Table allowed the teachers to see the children's progress and, thus, the result of the education they gave more quickly. Also, reporting new progress made them proud at the weekly meetings, resulting in regular assessment sessions. Further research on teachers' motivation for differentiation processes will illuminate the issue better.

The second issue was that teachers were concerned about children's emotional well-being. During the interviews, they often mentioned the significance of interacting with children in a caring manner when they were teaching mathematics. Their concern for children's feelings may result from the PD program's emphasis on supporting children's self-confidence. The findings showed that the teachers adapted the program's motto of "No child will leave the activity without any feeling of success." as a criterion while planning and implementing their activities.

The teachers' conscious efforts to create a classroom atmosphere conducive to learning for each child should have supported children's relationship with the teacher, the environment, and the subject. In this context, children who see that they have succeeded in mathematics will develop a positive perception of mathematics and have increased motivation to learn. Studies are needed to investigate this point further.

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